NOTICE: THIS MATERIAL MAY BE
COPYRIGHT LAW (TITLE 17 U.S. CODE)

OMICS

sity Press,

s." *Annals*

1959.

conomics

9 Dynamics and Land Use: The Case of Forestry

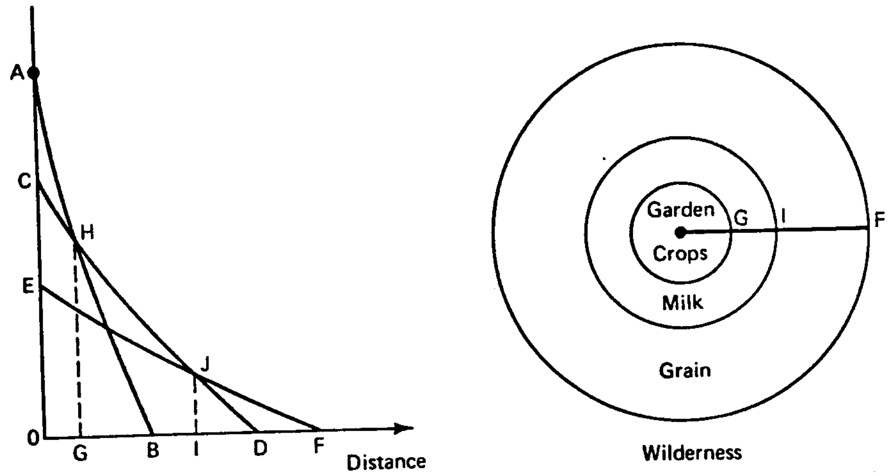
John Ledyard
Leon N. Moses

In 1836 Thunen published the classic volume in which he developed his theory of agricultural rent and land use.[1] He assumed a plane in which transport was a ubiquity and all land was of uniform quality. In the middle of this plane was a town or marketing center where the agricultural products that could be grown in the region were sold. The town and its agricultural hinterland were taken to be isolated from all other areas and surrounded by an uncultivated wilderness. Thunen formulated a model that determined a rational allocation of land to the alternative crops and the economic limit of cultivation beyond which the wilderness began. This model treated prices for products in the marketing center and the costs of transporting them from farms to the center as given. These costs were functions of distance. They also varied by product, some crops being more difficult to transport than others because of their greater bulk or perishability. Thunen assumed a fixed coefficient production function with a fixed yield per unit of land and fixed requirements of capital and labor for each crop. Finally, he assumed a wage rate, which he thought might decline with distance from the town, and a uniform return on capital.

With the structure of his model thus established, Thunen was able to derive what have become known as bid-rent functions. Each function pertains to a given crop. It shows the rent that land located at varying distances from the town would yield if devoted to that crop. This rent is the difference between on-the-farm gross revenue, which is the product of price at the town and yield, and the sum of labor, capital, and transport cost. With perfect foresight and competition, or perfect planning, each parcel of land is allocated to the use in which it yields the maximum rent and all land is thereby allocated in an optimal fashion. The wilderness area begins at that distance where land yields a zero rent.

Figures 9-1 (a) and 9-1 (b) illustrate the workings of the model for a three-crop system. Three bid rent functions are shown in 9-1(a). In order, AB, CD, and EF show the rent that would be yielded by land at varying distances from the center when devoted to garden crops, milk or pasture, and grain. The intersections of the bid rent functions determine the distances from the town at

This paper is based on a talk presented at a symposium, *The Economics of Sustained Yield Forestry*, College of Forest Resources, University of Washington, November 1974. The authors wish to acknowledge the very considerable benefit they have derived from a second paper presented at this symposium: Paul A. Samuelson, "Economics of Forestry in an Evolving Society."



(a) Bid-Rent Functions (b) Patterns of Land Use

Figure 9-1. Model for a Three-Crop System

which the three crops are grown, and the outer envelope, AHJF, of the bid-rent functions is the rent gradient. Thus, for example, land "at" the marketing center and to a distance OG from it, is devoted to the production of garden crops, and yields rent from OA to HG. As shown in 9-1(b), the model yields concentric rings, each of which is exclusively devoted to a given land use. OF is the limit of cultivation.

Thunen's theory of land use and rent evoked admiration among scholars for generations, but little was done to advance the model until recently when economists, geographers, and others adapted it to an urban setting. In modern versions Thunen's marketing center becomes the central business district (CBD) of a city. His crops became such urban land uses as finance, retailing, manufacturing, and housing. Instead of crops being shipped to the center, labor commutes. The object is still to explain how competition determines the price of space, which is shown to be a declining function of distance from the CBD. However, the tools of modern microeconomics have enriched the model and permitted a wider range of problems to be handled.[2] Thus, Vickrey and Solow have introduced congestion into a land use model.[3] In their work transport cost per unit shipped depends on the total volume of movement rather than being constant as in the Thunen model. Instead of taking the price of goods as given, Muth has developed a model in which prices are determined and goods as well as land markets are cleared.[4] Mills has introduced scale economies into the model, a feature which is essential to an understanding of urban development.[5] Beckmann and Koopmans,[6] Goldstein and Moses,[7] and Mills[8] have attempted to take into account the effect on location and land rent of activities that are interdependent since they use each others outputs as inputs.

While the tools of modern economic analysis have been used to significantly improve certain aspects of land use reasoning, there are other areas where the theory, as against certain empirical understandings, has not been advanced much beyond where Thunen left it. One such area is the effect of time on patterns of land use. To the author's knowledge there are no formal dynamic models that show how time and transport cost interact to determine rents, land uses, and intensities of cultivation at varying distances from a center. This is the subject of this chapter. We have introduced time and its capital theoretic implications into a model of land use in which the output is timber. The effect of time in such a model is of course opposite to that in something like urban housing. Over time a house deteriorates. The quality of the service it yields declines unless there is expenditure for maintenance.[a] At least up to the point where trees reach maturity, time has an opposite effect in forest land. Up to that point the yield from a tree or a stand of trees increases, so that time has a positive rather than a negative marginal productivity. We have chosen to develop our model of dynamic land use in a forestry context because there has been a lively debate on the issue of forestry management for many years. The nature of this debate is explained below.

It is interesting to note that Thunen himself was concerned with some of the dynamic aspects of land use. He considered alternative crop rotation systems, some of which would enrich the soil over time more than others, and some of which would exhaust the soil.[9] Thunen was also interested in the effect of time on the competition for land between forests and annual crops. He attempted to use precisely the same framework as in the remainder of his work, employing specified prices at the town, transport costs, etc., to determine which land would be devoted to commercial wood production for fuel and housing. However, as Samuelson has pointed out, Thunen and most other economists did not understand the capital theoretic aspects of the forestry case.[10]

Thunen assumed a 5 percent interest rate in his isolated state and then observed that there were forests where the annual increment in mass of the trees was 2.5 percent. In these circumstances he concluded that the woodlands would be destroyed and would not be replanted even if their gradual destruction raised the price of timber. He reasoned that each increase in price would simply increase the capital embodied in the timber stock and the owner of the forest

[a]Our statement that relatively little has been done to incorporate dynamic reasoning into land use models is not meant to suggest an absence of significant work on the subject. Many economists, planners, and others have been interested in how the quality of the housing stock changes over time and how the stock is filtered to lower income groups. For example, the National Bureau of Economic Research urban simulation model, Ingram, G.F.; Kain, J.F. and Ginn, J.R. with contributions by Brown, H.J. and Dresch, S.P. *The Detroit Prototype of the N.B.E.R. Urban Simulation Model.* New York: 1972 has a sub-model in which decisions are made on maintenance expenditures period by period. However, the complexities of the solution procedure rule out present value calculations and force the authors to adopt a set of ad hoc rules on such things as the net percentage of the stock in each residential area that can be filtered up or down in each period.

would therefore profit from felling the trees and investing at 5 rather than 2.5 percent. Thunen concluded that only a fall in the interest rate to 2.5 percent would halt the destruction of the woodlands. He then added that "... if the interest rate does not fall, and such an indispensable commodity as firewood is not to vanish from our earth, the governments will have to take steps to deprive citizens of their rights to dispose as they choose of their woods, forcing them to make do with only half the potential revenue from their forest property.[11] Thunen did understand that in the early years of development of a tree, or an entire forest, its mass might increase at much more than 5 percent per year and therefore that trees cut today might be replanted with young trees. However, he failed to incorporate this understanding into a steady state model.

Thunen's fear that the interest rate could lead to the destruction of forests still haunts foresters, particularly those who manage public forests. The Forestry Service of the U.S. Department of Agriculture, and the corresponding agencies in Canada and other wood-producing countries have adopted a policy known as maximum sustained yield forestry. Essentially the policy comes down to managing the forest so as to maximize the mean annual increment of wood.[12] That is, the forest is permitted to grow until average product with time is a maximum. This amount of timber is then cut each year and replaced with new trees. The influences of the interest rate and even of timber prices and costs of production are ignored in the sustained yield model.

Clearly what many foresters have not understood is that they are not managing forests. They are managing land which can be put to alternative uses, including the planting of new trees. What is needed is a policy based on a model that combines two things: (1) Thunen's conception of rent as it varies with distance and transport costs from a center; (2) Samuelson's capital theory reasoning of the impact of the interest rate and other costs on the steady state solution for any given parcel of forest land without regard to location and transport costs. This chapter attempts to develop such a model. We assume a center in the middle of a forest. This center is a town in which there is a wood processing mill. The price of timber at the mill is given. There are transport costs entailed in shipping timber to the mill and in sending labor out from the town to cut trees and plant new ones. There are other costs of production as well. The timber mill is assumed to operate under perfectly competitive conditions. The model determines the limit of economic cultivation of the forest for such a firm. There is a comparable concept in the forestry literature but it is not clearly defined and does not appear to be determined on the basis of economic considerations. The model shows the impact of transport costs on the length of time that trees are permitted to grow on land located at varying distances from the center. It also shows the impact of these costs on intensity of cultivation. The paper emphasizes the long-run steady state equilibrium rather than the path to equilibrium. The land use decisions a perfectly competitive firm would make are compared with those implied by the policy of maximum sustained yield forestry. We conclude that the latter is a sub-optimum policy.

An Idealized Forest[b]

Initially we ignore transportation costs on output and labor and consider a cyclical model of the forest in which at some time t, labor is used to initiate growth through clearing, planting, etc. At $t + T$, the timber is harvested and then labor is used to initiate a new growth cycle.[c] The available harvest at any date $t + T$ depends on the amount of labor used at t and the length of time, T, growth has occured.

Assume that forests grow in accordance with a biological growth law:

$$\dot{M} = f(M(T)) \tag{9-1}$$

where $M(T)$ is the biomass (broad-feet or some other measure) on a given land area at time T, and $\dot{M} \equiv dM/dt$ is the rate of growth of this biomass. Graphically, this growth law is represented in figure 9-2.

Here, $\bar{M}$ is the maximum amount of biomass which the land-area will support. It would be the biomass of a virgin forest. Labor inputs influence outputs because they determine the initial biomass from which growth occurs.[d]

$$M(0) = h(L) \tag{9-2}$$

where L is the amount of labor devoted, for example, to planting. We assume positive but diminishing returns to this type of effort. That is, $h'(L) > 0$ and $h''(L) < 0$.

The combination of biological law (9-1) and technology (9-2) give us our production function. In particular, given L, let $M(T,M_0)$ be the solution to (9-1) through $M_0 = h(L)$. The biomass available for harvest and sale at T, given the labor input, L, is simply

$$X_T = M(T, h(L)). \tag{9-3}$$

[b]The model is similar to Waggener's normal forest under full regulation with closed crown cover at each age.[13]

[c]Two additional uses of labor are ignored in this chapter. The first is the labor required to harvest the timber. This could be easily accounted for without changing the analysis by assuming a fixed amount of labor per biomass unit to be harvested. On this point see footnote e.

The second type are inputs used to thin, spray, etc., trees over the course of their development. Such labor increases the intensity of cultivation, and could act as a substitute for planting inputs. Inclusion of the former inputs in the model could affect our results if their impact on growth is large. On this point see footnote j.

[d]Labor is used here to denote all inputs needed to initiate the growth of a forest. i.e., nursery facilities, planting labor, etc.

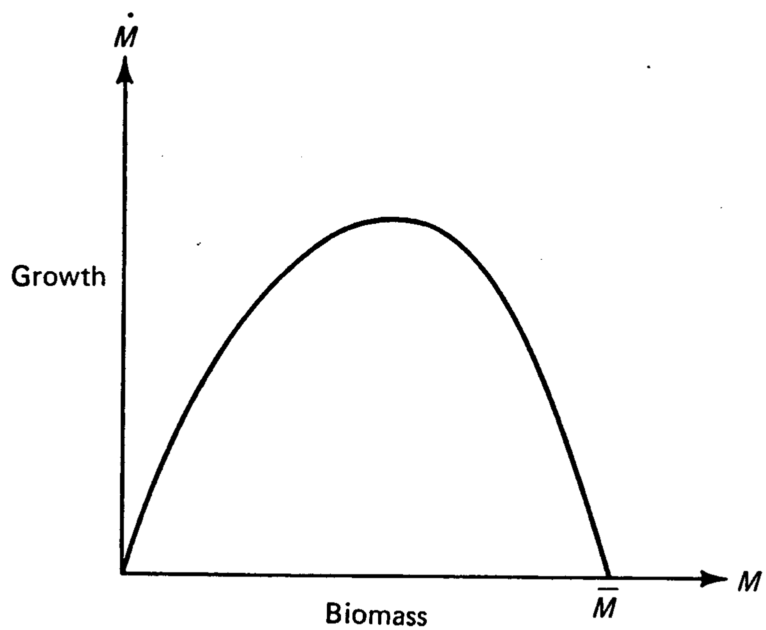


Figure 9-2. Biological Growth Law

In more familiar notation the production function is

$$X_T = F(T,L) \tag{9-3'}$$

where X_T is the output per land area at T.

The Competitive (Optimal) Solution Without Transport Costs

The forester who is confronted with perfectly competitive markets for output, labor, and land will choose T and L to maximize the present discounted value of profits. Thus, in steady-state competitive equilibrium L and T will be chosen to

$$\underset{T,L,}{\text{Maximize}}\ [pF(T,L)e^{-rT} - wL]\ (1 + e^{-rT} + e^{-2rT} + \ldots) \tag{9-4}$$

or

$$\text{Maximize } V(T,L) \equiv [pF(T,L)e^{-rT} - wL]\,(1 - e^{-rT})^{-1} \qquad (9\text{–}4')$$

where p is the price of lumber and w is the wage rate.[e]

In these competitive markets, the value or competitive purchase price of the land area utilized is

$$V^* = \underset{T,L}{\text{Max}}\; V(T,L)$$

and the instantaneous rental rate is $R^* = rV^*$. As Samuelson has indicated, an equivalent problem to (9–4) in competitive land markets, is to rent the land for the period of a single cycle.[14] The producer would then

$$\text{Maximize } pF(L,T)e^{-rT} - wL - R\int_0^T e^{-rT}dt = \Pi(R) \qquad (9\text{–}4'')$$

where R is set in competitive markets for land at its highest value, R^*, such that $\Pi(R^*) = 0$. The purchase price of land is $R^*/r = V^*$ as above.

At this point, we note three facts about optimal land use in our model. First, rents and optimal output are simultaneously determined since land is a variable factor of production through the decision variable T. Over the life cycle of one tree the owner of land should take opportunity cost into account even if there is no use for the land other than as a forest. This is true because a tree of age T should be viewed as competing for the land with newly planted trees. The opportunity cost of leaving a tree of age T on the land to grow another year is the present discounted value of profits foregone by not beginning future growth cycles on that date and by waiting until the next year to do so. In our model, the competitive rental, R^*, is the value of this foregone opportunity. The error that Thunen and many foresters have committed is that they have not taken this opportunity cost into account.

A second fact about optimal land use in our model is that if the maximum purchase price a forester should be willing to pay for land, V^*, is less than zero, then the land should not be used to produce timber. It should be left fallow or in a virgin state. This coincides with similar conclusions in traditional land use

---

[e] If b units of harvesting labor are required per unit biomass harvested, then (9–4) would read

$$\text{Max } [(p - wb)F(T, L)e^{-rT} - wL]\;(1 + e^{-rT} + \ldots).$$

The inclusion of harvesting labor would not change any of the qualitative properties of the model.

models. However, as we shall see it is not the conclusion implied in the policy of maximum sustained yield.

A third fact is that if there is an alternative use for the land which would yield a higher competitive purchase price than V^*, then the land should be used for that alternative and not for forests. Such a situation would exist if there were, for example, a one-year crop which yielded a profit on the land each year greater than $R^* (1 - e^{-r})/r = V^* (1 - e^{-r})$, the maximal one-year payment a competitive forester would be willing to pay.[f]

Optimal Land-use with Transportation Costs

As indicated earlier a single point, the mill, exists from which all labor must travel to work and to which all output must be brought for sale. Transport is ubiquitous and labor or output can be moved between any place and the mill at a per unit cost which depends only on the distance between the two locations. As in all land-use models of this type, rings of cultivation are determined within which all output and input decisions are identical.

Let $C_x(d)$ be the cost of shipping a unit of output a distance d to the mill[g] and let $C_L(d)$ be the round-trip cost of transporting labor this distance into the forested area. The competitive forester now chooses $T(d)$ and $L(d)$ to

$$\underset{T,L}{\text{Maximize}}\ V(T,L,d) \equiv [(p - C_x(d))Fe^{-rT} - (w + C_L(d))L]\ (1 - e^{-rT})^{-1}. \qquad (9\text{-}5)$$

In competitive land markets the instantaneous rental rate for land at a distance d from the mill is

$$R(d) = \underset{T,L}{\text{Max}}\ r \cdot V(T,L,d). \qquad (9\text{-}6)$$

We now turn to the task of describing how rentals, output, labor usage, and

---

[f]Alternatively, if Π' is the maximum profit yielded by a one year crop then the producer of that crop would be willing to pay $V' = \Pi'(1 + e^{-r} + e^{-2r} + \ldots) = \Pi'(1 - e^{-r})^{-1}$. If $V' > V^*$ then the land should be used for the one year crop. $V' > V^*$ holds when $\Pi'(1 - e^{-r})^{-1} > V^*$ or $\Pi' > V^*(1 - e^{-r})$.

[g]This includes the empty trip out and the loaded trip back.

harvesting times vary with the distance from the mill in steady-state competitive equilibrium. The necessary conditions for a solution to (9-5) or (9-6) are

$$[p - C_x(d)]\ (F_T - rF) - R = 0 \tag{9-7a}$$

$$[p - C_x(d)]\, F_L e^{-rT} - (w + C_L(d)) = 0 \tag{9-7b}$$

$$[p - C_x(d)]\, Fe^{-rT} - (w + C_L(d))L - R(1 - e^{-rT})r^{-1} = 0 \tag{9-7c}$$

The solution to (9-7) can readily be derived. From (9-3) we note that $F_T = \partial M/\partial T$, and from (9-1) that $\partial M/\partial T = f[F(T, L)]$. Thus $F_T = f(F)$ and equation (9-7a) simply requires that $f(F) - rF = R/(p - C_x)$. Figure 9-3 illustrates how F, and therefore T, can be determined. The first-order condition (9-7a) implies that the profit maximizing output is either X^1 or X^2 in figure 9-3(a). The second-order conditions indicate that X^2 is the appropriate choice.

To demonstrate this result we note that A^*, the matrix of second partial derivatives of the profit function (9-4″), must be negative semi-definite where

$$A^* = (p - C_x) \begin{bmatrix} F_{TT} - rF_T & F_{TL} - rF_L \\ (F_{LT} - rF_L)e^{-rT} & F_{LL}e^{-rT} \end{bmatrix}.$$

By applying comparative dynamics to the growth laws (9-1) and (9-2) it can be

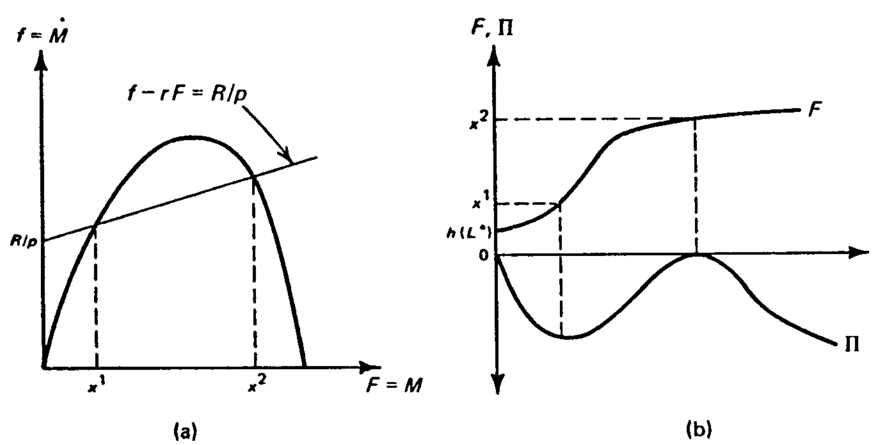


Figure 9-3. Profit Functions and Maximization

shown that $F_L = e^{g(t)} h'(L)$, where $g(T) = \int_0^T f'[F(t,L)]\,dt$. It follows that $F_{TL} = f'F_L = F_{LT}$. Also, $F_{TT} = f'F_T$. Thus A^* can be written as

$$(p - C_x)\begin{bmatrix} (f' - r)f & (f' - r)F_L \\ (f' - r)F_L e^{-rT} & F_{LL} e^{-rT} \end{bmatrix}.$$

Since A^* must be negative semi-definite, $f' - r \leqslant 0$. Thus, in figure 9-3(a), X^2 is the profit-maximizing output. Figure 9-3(b) illustrates the growth curve $F(t, L^*)$ and the equilibrium single-cylce profit function

$$\Pi(t) = Fe^{-rT} - (w/p)L^* - (R/p)(1 - e^{-rT})r^{-1}$$

for the equilibrium quantity of labor, L^*.

Standard comparative statics analysis can be applied to (9-7) to discover how T, L, and R change as the distance to the mill changes by solving the following system of three equations:

$$\left[\begin{array}{cc|c} \multicolumn{2}{c|}{\multirow{2}{*}{A^*}} & -1 \\ & & 0 \\ \hline 0 & 0 & -(1 - e^{-rT})r^{-1} \end{array}\right] \begin{bmatrix} dT \\ dL \\ dR \end{bmatrix} = \begin{bmatrix} C_x'(F_T - rF) \\ C_x'F_L e^{-rT} + C_L' \\ C_x'Fe^{-rT} + C_L'(L) \end{bmatrix} d(d). \qquad (9\text{-}8)$$

If A^* is assumed to be negative definite, system (9-8) can be solved and it can be shown that

$$dR/dd = -(1 - e^{-rT})^{-1} r\,[C_x'Fe^{-rT} + C_L'L], \qquad (9\text{-}9)$$

$$\begin{aligned} dT/dd = |A|^{-1}(p - C_x)\Big\{ & C_x'e^{-rT}[-F_{LL}(1 - e^{-rT})(f - rF) \\ & + (f' - r)F_L(1 - e^{-rT})r^{-1}F_L + F_{LL}e^{-rT}F] \\ & + C_L'[(f' - r)F_L(1 - e^{-rT})r^{-1} + F_{LL}e^{-rT}L]\Big\}. \end{aligned} \qquad (9\text{-}10)$$

$$\begin{aligned} dL/dd = |A|^{-1}(p - C_x)\Big\{ & C_x'e^{-rT}[(f' - r)F_L(1 - e^{-rT})r^{-1}(f - rF) \\ & - (f' - r)F_T(1 - e^{-rT})r^{-1}F_L - (f' - r)F_L F e^{-rT}] \\ & + C_L'[-(f' - r)F_T(1 - e^{-rT})r^{-1} - (f' - r)F_L e^{-rT}L\Big\}. \end{aligned} \qquad (9\text{-}11)$$

We now make the eminently reasonable assumption that total transportation costs increase with distance. That is, we assume that C_x' and C_L' are positive.

In addition a positive interest rate, r, is assumed. From these assumptions and equation (9-9) it immediately follows, as one would have guessed, that rent per unit of land is lower the further it is located from the shipping point. That is, $R'(d) = dR/dd < 0$.

Equations (9-10) and (9-11) can be rewritten by recognizing that

$$Fe^{-rT} - (f - rF)(1 - e^{-rT})r^{-1} = Fe^{-rT} - [R/(p - C_x)]\,(1 - e^{-rT})r^{-1}$$
$$= [w + C_L/(p - C_x)]\,L.$$

One then observes that

$$T' = |A|^{-1}(p - C_x)\Big\{C_x' e^{-rT}[F_{LL} rL(w + C_L/p - C_x) + (F_L)^2(1 - e^{-rT})(f' - r)] + C_L'[F_{LL} Le^{-rT} + (f' - r)F_L(1 - e^{-rT})r^{-1}]\Big\} \qquad (9\text{–}10')$$

and

$$L' = |A|^{-1}(p - C_x)\Big\{C_x' e^{-rT}[(f' - r)F_L(-L)[w + C_L/p - C_x] - (f' - r)F_T F_L(1 - e^{-rT})r^{-1}] + C_L'[-(f' - r)(F_T[1 - e^{-rT}]r^{-1} - e^{-rT} LF_L(f' - r)]\Big\}. \qquad (9\text{–}11')$$

While it may not be obvious, relations (9–10′) and (9–11′) allow us to conclude that $T' > 0$ and $L' < 0$.[h] In other words, *the optimal management of forests and land requires that the further away land is from the mill, the less labor one should employ in planting, and the longer one should allow the forest to grow before harvesting.*[i,j]

---

[h] One observes that $|A| < 0$ since A^* negative definite implies $|A^*| > 0$. Also $f' - r < 0$, $F_L > 0$, $F_T > 0$, $F_{LL} < 0$, and $(w + C_L)/(p - C_x) > 0$.

[i] The fact that T increases as the net price, $(p - C_x)$, decreases is the "Ricardo effect" of capital theory.

[j] The conclusions so far reached have been based on the assumption that labor inputs only affect output by determining the initial biomass. Suppose, on the other hand, that the forester can also influence rates of growth by applying inputs over time for such activities as thinning, spraying, etc. That is, $\dot{M} = f(M, L)$. Some of the conclusions reached in the chapter may not hold if $\partial f/\partial L$ is large and positive when evaluated at $M = F(T^*, L^*)$, the competitive output level. $R' < 0$ will still hold. However, the expression for T' in (9–10′) will have an additional term $f_L|A|(p - c_x)\left\{C_x' F_L(1 - e^{-rT}) + C_L'(1 - e^{-rT})r^{-1}\right\}$ which is negative. The expression for L' in (9–11′) will have the additional term $F_L|A|(p - C_x)\left\{- C_x' e^{-rT} L\right.$

It is also true that output per land unit at the harvest date, $X_T = F(T, L)$, increases as the distance the land is located from the mill increases, since

$$dF/dd = [d(R/(p - C_x))/dd]\ (f' - r)^{-1} > 0.^{k} \qquad (9\text{-}12)$$

Finally, it is relatively easy to show that $R'' = dR^2/dd^2 > 0$ if per unit transportation costs do not increase at an increasing rate with distance: that is, if C_x'' and C_L'' are less than or equal to zero. This follows since

$$R'' = (\partial R'/\partial T)T' + (\partial R'/\partial L)L' + \partial R'/\partial d, \partial R'/\partial T > 0, \partial R'/\partial L < 0$$

$$\text{and } \partial R'/\partial d > 0.$$

A representative bid-rent curve, $R(d)$, is graphed in figure 9-4. The number d_c determines the limit of the working circle. Beyond d_c it is unprofitable and non-optimal to engage in commercial forestry.

Land Use Under Sustained Yield Policy

If the policy of maximum sustained yield is interpreted literally, the forester chooses a strategy which yields the highest sustainable output from a normal forest while maintaining a constant vintage structure of trees in the forest.[l] In terms of our model the forester chooses T and L to

$$\max_{L,T} F(L,T)/T. \qquad (9\text{-}13)$$

This strategy determines how a particular land unit is to be managed. However, it does not define the size of the working circle, which is to say the amount of land that is to be managed. A search of the forestry literature failed to reveal any specific policy with respect to the size of the working circle for public lands. Therefore we shall examine the implications of two alternative definitions that

---

$[w + C_L/p - C_x] - C_L' e^{-rT_L}\}$ which will be positive. The expression for F' in (9-12) will have the additional term $-f_L L'(f' - r)^{-1}$ which could be negative if $L' < 0$.

If $f_L < -(f' - r)F_L$, these additional terms will not change the qualitative results reported in the text. Whether labor can have a large enough impact on the rate of growth of a mature forest to cause $f_L > -(f' - r)F_L$ is an empirical issue which cannot be decided in this chapter.

[k] Since L declines with distance, fewer trees are planted. Since T increases, each tree matures longer. The fact that X increases means that the increased maturing more than compensates for the decreased planting. Thus, forests are denser at harvesting date the further one moves from the mill.

[l] This is the maximum mean annual increment discussed by Waggener.[15]

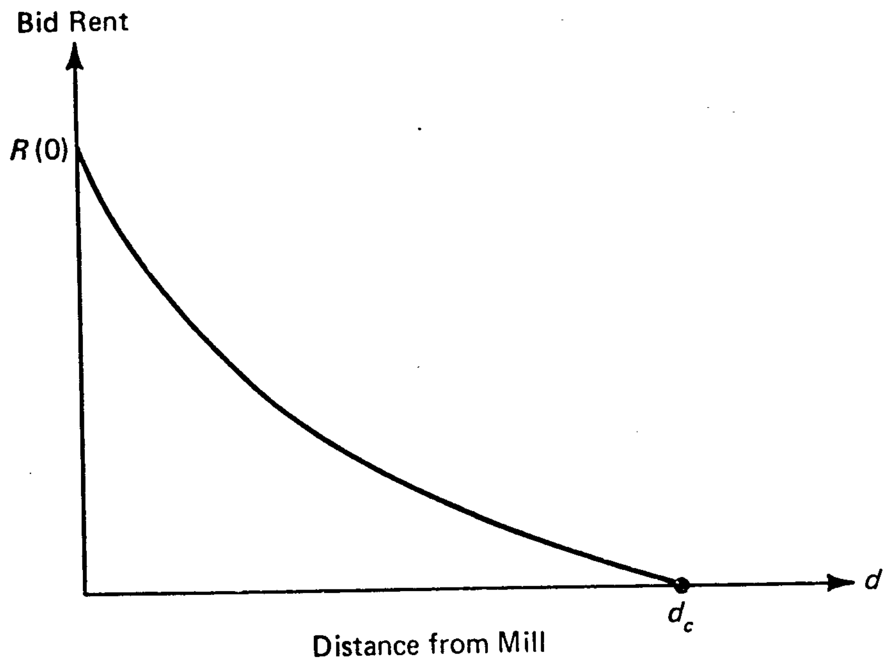


Figure 9-4. A Representative Bid-Rent Curve

are in keeping with the broad philosophy of sustained yield. As will be shown the first implies a negative annual cash flow.

Equation (9-13) determines the date on which trees should be harvested in order to achieve maximum sustained yield. If contracts are then arranged on a competitive basis with private firms, the maximum amount they would be willing to bid for the privilege of harvesting an acre of land is the revenue they can make from their operation. In our model this is $p - C_x(d)$ per unit biomass harvested from land d units away from the mill.[m] Thus, as long as $p - C_x(d) \geqslant 0$ private firms will be willing to harvest trees from public lands. Under competitive bidding the extensive limit of cultivation will occur at distance d_M from the mill, where $p - C_x(d_M) = 0$. It should be obvious that d_M is greater than the competitively determined extensive limit of cultivation, d_c. A land use policy based on this definition of the working circle generates an annual cash flow of

$$[(p - C_x)F(T,L) - (w + C_L)L]/T. \qquad (9\text{-}14)$$

This will be negative at d_M and at some other locations within d_M of the mill.

[m]If harvesting labor is included in the model, as in footnote e, the revenue per unit harvested at distance d is $p - (w + C_L(d)b - C_x(d)$.

The negative cash flow results from the fact that the private firms ignore the cost of planting new trees.

The second interpretation of the working circle is based on the assumption that the managers of public forests wish to avoid the above losses. They then choose T and L to maximize net sustained cash flow as defined by (9-14). A land use policy consistent with this goal utilizes all land which yields a non-negative cash flow. That is, all land up to a distance d_N from the mill will be used, where d_N is defined by the following equation:

$$\max_{T,L} \left[(p - C_x(d_N))F(L,T) - (w + C_L(d_N))L\right]/T = 0.$$

This approach leads to a less extensive use of land; that is, $d_N < d_M$.

It can also be shown that d_N is greater than d_c, the competitive and optimal extensive limit. To demonstrate this conclusion, we first note that the values of T and L generated by this policy are equivalent to the competitive result if the interest rate, r, equals zero.[n] It can be shown that as r decreases, the competitive value of land increases;[o] that is, $dV^*/dr < 0$.[p] Finally, when the interest rate is zero, competitive rent, R_0, is equal to the maximal value of the net sustained cash flow. Hence d_N, the distance at which $R_0(d_N) = 0$, is larger than d_c (see figure 9-5). Thus, even a policy of maximizing net sustained cash flow leads to a more extensive use of land for timber production than is socially desirable, assuming no externalities.

Summary and Conclusions

This chapter contains a model that combines Thunen's theory of rent and land use, which depends very considerably on transport costs, with capital the-

[n] The reader should recall that the competitive choices were made to maximize $pFe^{-rT} - wL - R\int_0^T e^{-rT}dt$. When $r = 0$ this is equivalent to maximizing $pF - wL - RT$, where R satisfies Max $[pF - wL - RT] = 0$. Thus when $r = 0$, L, and T are chosen under competition such that $(pF - wL)/T$ is a maximum.

[o] $dV/dr = d\left(\frac{R}{r}\right)/dr = r^{-2}\,[rdR/dr - R]$.

However,

$$dR/dr = -(1 - e^{-rT})^{-1} r[T(p - C_x)Fe^{-rT} + T(R/r)e^{-rT} - R(1 - e^{-rT})r^{-2}] =$$
$$-r(1 - e^{-rT})^{-1} Te^{-rT}\left[(p - C_x)F + \frac{R}{r}\right] + \frac{R}{r}.$$

Hence,

$$rdR/dr - R = -r(1 - e^{-rT})^{-1} Te^{-rT}[(p - C_x)F + R/r],$$

which is less than zero.

[p] Despite the fact that $dV/dr < 0$ we cannot conclude that $dR/dr < 0$ since $dR/dr = d(rV)/dr = rdV/dr + R$. The last may be positive even if $dV/dr < 0$. At the extensive limit, however, $V = R = 0$. Thus, at these locations $dV/dr < 0$ and $dR/dr < 0$.

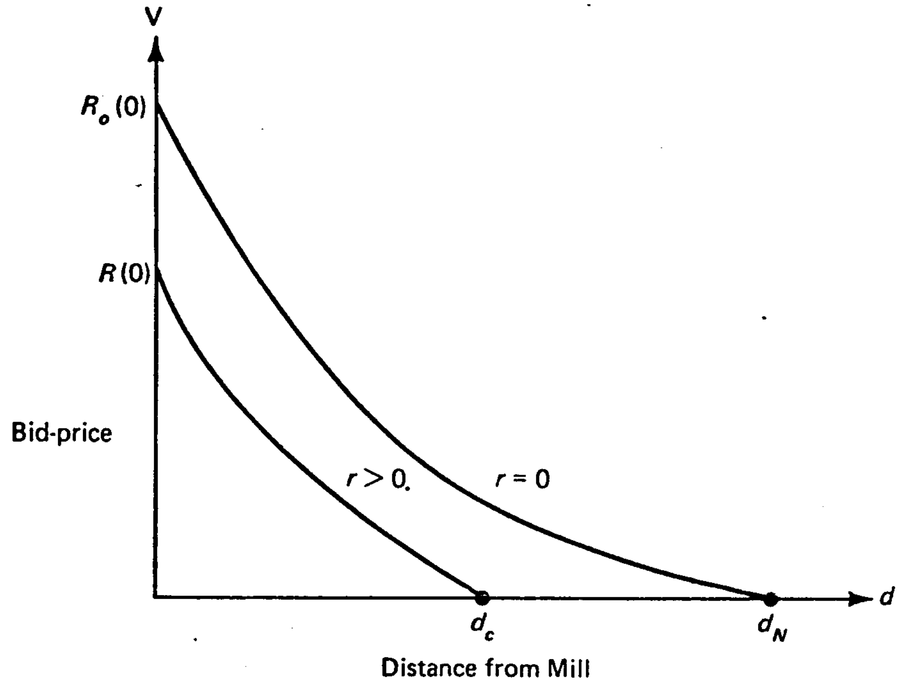


Figure 9-5. The Optimal and Zero Interest Rate Bid Rent Curves for Timber Land

ory. The model was developed for the case of forestry and focused on land use in long-run steady state competitive equilibrium. Among the results derived was the obvious one that the rental value of forestry land declines with distance from the processing mill, which was also treated as the market. In addition, as distance from the mill increases less labor is employed in clearing and planting each acre of land, and trees are permitted to grow for a longer period of time. That is, as distance from the mill increases time is substituted for labor. Finally, the farther an acre of land is from the mill, the greater is output at harvest time because the effect of time more than compensates for the smaller quantity of labor employed.

The chapter also examined some public policy issues involved in the management of public forest lands. In particular, the results of the competitive model were compared with those implied by the policy of sustained yield. We found that the latter entails a more extensive use of land for timber production. In other words, with the competitive model the working circle is smaller and more land is left in virgin forest. Sustained yield can therefore be viewed as a sub-optimum policy so far as the extensive limit of cultivation is concerned. Samuelson reached the conclusion that sustained yield allows trees to grow too long and is, therefore, a sub-optimal policy for any given acre of land, without regard to location.[q]

The conclusion that the policy of sustained yield leads to sub-optimum results is based on the observation which should be familiar to all economists, that in the absence of externalities and in the presence of accurate price expectations, competitive markets will lead decisionmakers to follow policies which are socially optimal. We do not argue that all forestry management should be turned over to the private sector and be subject to market regulation but only that the competitive outcome can be used as a benchmark to judge alternative land use policies.

Certain externality arguments can be made with regard to forests, though it should be noted that these arguments are usually made to justify the existence of forests rather than timber production. We are aware of two such arguments. One is that forested areas benefit the ecological environment through their impact on erosion and flood control, cleaner air, etc. The second is that forests are beautiful and should therefore exist. We have demonstrated that the policy of sustained yield leads to a more extensive use of land for timber production and less virgin forest than the competitive solution. In this respect consideration of externalities therefore reinforces the conclusion that sustained yield is a sub-optimum policy. On the other hand, under the competitive solution trees close to the mill may be harvested at a younger age than under sustained yield. If older trees yield more in the way of flood control or are considered more aesthetically pleasing, the externality argument may go against our conclusion. Whether it does so or not depends on the value of having more virgin forest located at a distance from the mill relative to the value of having older trees close to the mill.

---

[q]Our model leads to the conclusion that the biomass at harvest time is greater under a policy of sustained yield than under competitive conditions. From (9–7a) it follows that

$$(p - C_x)\partial F/\partial r = [(p - C_x)F + \partial R/\partial r](f' - r)^{-1}.$$

Substituting for $\partial R/\partial r$, from footnote o, yields

$$(p - C_x)\partial F/\partial r(f' - r) = [(p - C_x)F + R/r]\ [1 - rT(1 - e^{-rT})^{-1}e^{-rT}].$$

But

$$1 - rT(1 - e^{-rT})^{-1}e^{-rT} > 0$$

when $rT > 0$ and equals zero when $rT = 0$. Thus, since

$$f' - r < 0,\ \partial F/\partial r < 0.$$

We cannot conclude that this result is unambiguously due to a longer growing cycle, since it is possible for a fall in the interest rate to lead to a shorter growing cycle, with the greater biomass harvested being due to increased labor. However, if there is little substitutability between labor and time, a condition that seems empirically valid, then our model yields the Samuelson result.

Notes

1. Heinrich Johann Von Thunen, *Von Thunen's Isolated State*, trans. by Wartenberg, Carla M., ed. by Peter Hall (Edinburgh: Pergamon Press, 1966).
2. For a review of the recent literature see G.S. Goldstein and L.N. Moses, "A Survey of Urban Economics," *The Journal of Economic Literature* 11 (June 1973).
3. Robert M. Solow and William S. Vickrey, "Land Use in a Long Narrow City," *Journal of Economic Theory* 3 (December 1971): 430–447.
4. Richard F. Muth, "Economic Change and Rural-Urban Land Conversions," *Econometrica* 29 (January 1961): 1–23.
5. Edwin S. Mills, "An Aggregative Model of Resource Allocation in Metropolitan Area," *American Economic Review* 57 (May, 1967).
6. T.C. Koopmans and M. Beckmann, "Assignment Problems and the Location of Economic Activities," *Econometrica* 25 (1957): 53–76.
7. G.S. Goldstein and L.N. Moses, "Interdependence and the Location of Economic Activities," *Journal of Urban Economics* 2 (1975): 63–84.
8. E.S. Mills, *Studies in the Structure of the Urban Economy* (Baltimore: The Johns Hopkins Press, published for Resources for the Future, 1972).
9. Von Thunen, *Thunen's Isolated State*, Chapter 18.
10. Paul A. Samuelson, "Economics of Forestry in an Evolving Society," paper delivered at a symposium, *The Economics of Sustained Yield Forestry*, College of Forest Resources, University of Washington, November 1974.
11. Von Thunen, *Von Thunen's Isolated State*, p. 119.
12. For a discussion and review of this policy see T.R. Waggener, "Some Economic Implications of Sustained Yield as a Forest Regulation Model," University of Washington, Forestry Paper Contribution No. 6, 1969.
13. Ibid.
14. Samuelson, "Economics of Forestry."
15. Waggener, "Some Economic Implications," p. 9.